Presented at the 17th Annual Convention of the Global Awareness Society International, May 2008, San Francisco, CA, USA

Investigating a Motivating Theme for Elementary Mathematics and Statistics Courses

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Abstract

Teaching introductory mathematics and statistics to non-majors has always been a challenge to instructors. Here the challenge lies in how to tie the material to students' everyday concerns or interests so that the subject matter becomes relevant and useful. Some recent research show that the applied quantitative methods taught within a context familiar or of interest to students increases their desire to learn and their tolerance to deal with complex concepts. With these in mind, I am planning to investigate an idea for motivating students in introductory mathematics and statistics classes. The motivating theme is the quantitative methods involved in studying and analyzing alcohol related data and information. The goal is to develop lesson plans for teaching several classical topics taught in introductory mathematics and statistics courses.

Information for Lessons

Drinking is considered a part of the college life by many students. As such data and information related to alcohol use and abuse may be utilized as a theme in introductory mathematics and statistics classes. For example, students often wonder how much one can drink to stay below the limit, or after consumption of a certain amount of the alcohol how long one should wait before driving. Some wonder about effect of factors such as weight, gender, race, etc in alcohol tolerance. The answers to these questions involve quantitative methods that can be used both for teaching and learning. The following presents examples of data and information useful for developing lessons.

Alcohol and Mathematics

In recent years, there has been a considerable amount of scientific research on the effects of alcohol on memory, reflexes, coordination, and depth perception as well as a host of other cognitive and psychomotor processes. Formulation of such research questions requires quantification of the amount of alcohol in the blood. This has led to the introduction of blood alcohol levels as a percentage of alcohol per volume of blood.

A blood alcohol level (BAC) of 0.10 is defined as 1 gram per kg of blood, meaning that alcohol is 0.10%, or one one-thousandth, of the blood. For example, a person weighing 150 pounds with 34 grams of alcohol in the body, the amount of alcohol from two and a half beers would have a 0.08 BAC equal to 0.08. The same person with 210 grams of alcohol in the body would have a 0.50 BAC.

Quantification of blood alcohol level helps investigation of many other questions and opens the door for introduction of other concepts found in elementary mathematics and related subjects. For example, it has been demonstrated that the probability of being involved in an alcohol related accident increases as the BAC increases. Estimates show that an individual with a BAC between 0.10 and 0.14 is 48 times more likely to be involved in a motor vehicle accident than an individual who has not been drinking. This can be used in discussion of probability and its definitions.

The BAC of an individual is determined by three primary factors: body weight, amount of alcohol consumed, and the amount of elapsed time from first drink until a breath and/or blood sample is taken. As an example, a 180-pound man who consumes seven drinks over a three-hour period of time will have a BAC of around 0.10. In contrast, a 110-pound man who consumes the same seven drinks over three hours will have a BAC of around 0.20. One drink, which is defined as one ounce of liquor, five ounces of wine or one 12-ounce beer, consists of approximately 14 grams of alcohol. Note that all these information and data can be used for developing lesson plans. The following is a specific example of alcohol-related

information useful for teaching functions, an important concept in elementary mathematics.

After a drink, the alcohol begins to enter the bloodstream almost immediately and the blood-alcohol level rises rapidly. Once the person stops drinking, her natural metabolic processes slowly eliminate the alcohol and blood-alcohol level begins to fall. So, how long must one wait after drinking for safe driving?

Experiments show that after drinking 6 beers, 4 glasses of wine or 4 shots of liquor, the blood-alcohol level typically rises to 0.11 gram per deciliter of blood. Thereafter alcohol is eliminated at the rate of 0.02 gram per hour. This relationship can be described by an important mathematical tool known as function. Functions have different types and can be classified according to their properties. One important classification is linear versus nonlinear. To introduce these, one can refer to the fact that, many substances are eliminated from the bloodstream *nonlinearly*, but alcohol is eliminated *linearly*.

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Alcohol and Statistics

Many popular newspapers publish statistics regarding alcohol, especially in relation to college students. For example, in March 2002, USA Today published an article titled “College drinking kills 1,400 a year, study finds.” The article was based on a research by Ralph Hingson of the Boston University.

In addition to the death toll, the report included statistics such as: 500,000 college students were injured while under the influence of alcohol; 600,000 were assaulted; 70,000 were the victims of sexual assault; 400,000 had unsafe sex; 25 percent had academic problems; and 150,000 have alcohol-related health problems or tried to commit suicide. The questions regarding the way these statistics were obtained can be used for teaching basic concepts of statistics such as sampling, estimation, etc. together with their use and abuse.

The use of statistics based on such research or investigations is clear. To see how they may be used to reveal the abuse of statistics, consider the following

example. Hingson has obtained 1400 deaths using the following approach. There are about 25.5 million 18- to 24-year-olds living in the U.S., according to U.S. Census data. Thirty-one percent of this age group are enrolled as full or part-time students in two-or four-year colleges. The number of alcohol-related motor vehicle crash deaths among 18-24 year olds during 1998 is 3,674; 31 percent of this figure is 1,138. Similarly, applying the 31 percent factor to the 991 alcohol-related, non-traffic deaths among 18- to 24-year-olds in 1998 results in an additional 307 deaths. Adding the 307 and 1,138 figures equals the alleged 1,445 alcohol-related deaths annually among college students.

When estimating this number, Hingson relies on a key, but unsupported assumption. It does not automatically follow that college students constitute 31 percent of alcohol deaths simply because 31 percent of 18- to 24-year-olds are college students. This reasoning is equivalent to assuming that because women constitute about half the population, they commit half of all crimes. In fact, men commit more than 75 percent of crime. This type of examples can be used to develop lessons regarding misuses of statistics.

The definition of what constitutes an “alcohol-related” death is another problem useful for teaching concepts such as hypothesis testing. In short, there are plenty of opportunities that we hope to explore and use as teaching tool.

Methodology

The method we plan to adopt may be explained by a specific example. Alcohol is absorbed into the body primarily through the lining of the stomach. For this reason, a BAC peak is usually reached within 20 minutes of the last drink. In general, chemicals in the blood stream are either eliminated by the kidneys or they are being broken down by enzymes from the liver. Kidneys tend to eliminate a certain proportion of chemicals during each time period. If the person has x gram of alcohol in the body ax gram of it will be eliminated over the next hour where a is a constant. Thus if x is the amount of alcohol in the body at the beginning of an hour

and *y* is the amount of alcohol eliminated during the next hour, the following relation holds

$$y = ax$$

This is the equation of a line that passes through the origin. Since the percent of alcohol removed from body per hour can vary greatly from person to person, this can be used to teach concepts such as slope and lines with different slopes.

The liver eliminates chemicals by breaking them down with enzymes. However, the liver may not break down a constant proportion of it each hour. Instead, the percent of the chemical being broken down can depend on the amount of it in the body, which is the case for alcohol. In fact, as the amount of alcohol in the body increases, the proportion of the alcohol the body eliminates decreases. For alcohol, the proportion *z* of the alcohol broken down in a given hour can be approximate using the following relation.

$$z = 10/(4.2 + x)$$

where *x* is the number of grams of alcohol in the body at the beginning of the hour. This is an example of what is called capacity-limited metabolism, in which the amount of the chemical metabolized depends on the amount of the chemical in the body. This is an equation of specific function and can be used to teach concepts such as graphing a function, derivatives (decrease per unit of *x*). Note that this formula is an average. For any individual person, the numbers 10 and 4.2 may vary considerably.

The examples presented are a sample of what we plan to use for developing lessons of basic algebra. Activities of the kind presented below will be included in the lessons.

1. Find the proportion of alcohol eliminated from the body during the next hour if there are 14, 28, and 42 grams of alcohol in the body at the beginning of the hour. (One drink consists of approximately 14 grams of alcohol.)

2. Let *x* represent the amount of alcohol in the body at the beginning of an hour. Let *y* represent the amount of alcohol eliminated from the body during that hour. To find the amount of alcohol eliminated during that hour, we may multiply the proportion eliminated by the amount in the body at the beginning of the hour, giving the formula

$$y = 10x/(4.2 + x)$$

Find how much alcohol is eliminated during an hour if the body began with 14, 28, 42 grams? Compare your results to the results of problem 1. This demonstrates that as the amount of alcohol, *x*, in the body goes up (from 14 to 28 to 42 grams), the proportion, *z*, of alcohol eliminated decreases, but the actual amount of alcohol eliminated, *y*, increases. This is a good example of increasing and decreasing functions, another classification used in algebra.

In summary, the process of the elimination of alcohol from the body serves as an introduction to a study of functions in general and rational functions in particular at an intermediate algebra level. The focus on lesson can be the graphs of the functions with emphasis on interpretation of the horizontal and vertical asymptotes in the context of elimination of alcohol from the body. Another lesson can focus on algebraic manipulation of the rational functions, solution of equations with rational expressions, realistic domain of a function, inverse functions, all based on formulations discussed earlier.

Questions regarding weight and gender can serve as an introduction to statistical test of hypothesis and also regression and correlation analysis. These possibilities will also be investigated.